

Shota Ohnishi · Yoichi Ikeda · Hiroyuki Kamano · Toru Sato

Signature of strange dibaryon in kaon-induced reaction

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Abstract We examine how the signature of the strange-dibaryon resonances in the $\bar{K}NN - \pi\Sigma N$ system shows up in the scattering amplitude on the physical real energy axis within the framework of Alt-Grassberger-Sandhas (AGS) equations. The so-called point method is applied to handle the three-body unitarity cut in the amplitudes. We also discuss the possibility that the strange-dibaryon production reactions can be used for discriminating between existing models of the two-body $\bar{K}N - \pi\Sigma$ system with $\Lambda(1405)$.

Keywords strange dibaryon · AGS equations · point method

1 Introduction

The structure of $\Lambda(1405)$ with spin-parity $J^\pi = 1/2^-$ and strangeness $S = -1$ has been studied for a long time. In the constituent quark model, $\Lambda(1405)$ might be considered as p-wave excited state with uds quarks. However, since the mass of $\Lambda(1405)$ is about 30 MeV below the $\bar{K}N$ threshold, it has also been suggested that $\Lambda(1405)$ is the s-wave $\bar{K}N$ quasi bound state due to the strongly attractive s-wave interaction of the $\bar{K}N$ system with $I = 0$ [1]. Akaishi and Yamazaki suggested that this strong attraction will produce a new type of nuclei, the deeply bound kaonic nuclei [2]. The simplest deeply bound kaonic nuclei are the strange dibaryon, which are the resonances in the $\bar{K}NN - \pi\Sigma N$ system. The strange-dibaryons will give a baseline for the systematic investigation of such deeply bound kaonic nuclei, because the many body dynamics can be treated accurately. The strange dibaryon resonances have been studied with the Alt-Grassberger-Sandhas (AGS) equations [3, 4] and with the variational method [5, 6, 7] using the phenomenological meson-baryon interactions [3, 5, 6] or interactions based on the effective chiral Lagrangian [4, 7]. All the analyses suggest the existence of the strange-dibaryon resonances.

The strange dibaryon resonances can be produced by photon- or kaon-induced reactions on light nuclei such as d and ^3He , and the signal of the resonances may be observed in the invariant mass and/or missing mass distributions of the decay products. Theoretical studies of the kaon-induced reactions have been done by Koike-Harada and Yamagata *et al.* within the optical potential approach [8, 9].

In this contribution, we present how the signature of the strange-dibaryon resonances in the $\bar{K}NN - \pi\Sigma N$ system shows up in the three-body scattering amplitude obtained by solving AGS equations on the physical real energy axis, which is the basic ingredient to calculate the cross sections for strange-dibaryon production reactions measured in the experimental facilities such as J-PARC.

S. Ohnishi · T. Sato
Department of Physics, Osaka University, Osaka 560-0043, Japan
E-mail: sonishi@kern.phys.sci.osaka-u.ac.jp

Y. Ikeda
Department of Physics, Tokyo Institute of Technology, Tokyo 152-8551, Japan

H. Kamano
Research Center for Nuclear Physics (RCNP), Osaka University, Osaka 567-0047, Japan

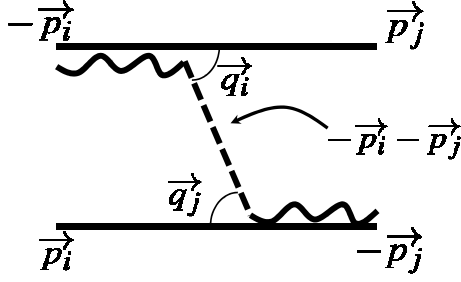


Fig. 1 One particle exchange interaction $Z_{i,j}(\mathbf{p}_i, \mathbf{p}_j, W)$.

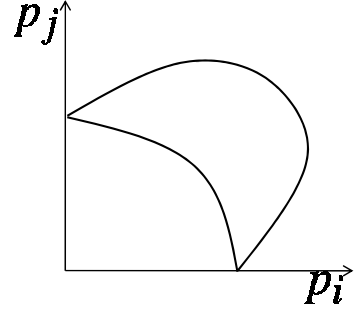


Fig. 2 The moon-shape singularities. Solid curve shows the momenta (p_i, p_j) where Z has logarithmic singularity.

2 Three-body Scattering Equations

2.1 AGS equations

The coupled channel equation for the $\bar{K}NN - \pi\Sigma N$ coupled channel system is given by the AGS equation

$$X_{i,j}(\mathbf{p}_i, \mathbf{p}_j, W) = (1 - \delta_{i,j})Z_{i,j}(\mathbf{p}_i, \mathbf{p}_j, W) + \sum_{n \neq i} \int d\mathbf{p}_n Z_{i,n}(\mathbf{p}_i, \mathbf{p}_n, W) \tau_n(W - E_n) X_{n,j}(\mathbf{p}_n, \mathbf{p}_j, W), \quad (1)$$

with the separable approximation for the interaction V

$$V(\mathbf{q}', \mathbf{q}) = \lambda g(\mathbf{q}')g(\mathbf{q}). \quad (2)$$

Here $X_{i,j}(\mathbf{p}_i, \mathbf{p}_j, W)$ is the quasi two-body amplitudes with the particle i (j) as the spectator in the final (initial) state; the energy W contains the infinitesimal positive imaginary part, $W = W' + i\epsilon$ with a real W' and a infinitesimal positive ϵ , resulting from the boundary condition of the scattering problem. The driving term $Z_{i,j}(\mathbf{p}_i, \mathbf{p}_j, W)$ for the s-wave depicted in Fig.1 is the particle-exchange interaction given by

$$Z_{i,j}(p_i, p_j, W) = 2\pi \int_{-1}^1 d(\cos \theta) \frac{g(q_i)g(q_j)}{W - \frac{p_i^2}{2m_i} - \frac{p_j^2}{2m_j} - \frac{p_i^2 + p_j^2 + 2p_i p_j \cos \theta}{2m_k}}, \quad (3)$$

where $\cos \theta = \hat{\mathbf{p}}_i \cdot \hat{\mathbf{p}}_j$. Because of the three-body propagator in the integrand of this equation, the interaction $Z_{i,j}(p_i, p_j, W)$ has logarithmic singularities in (p_i, p_j) plane known as the moon-shape singularities shown in Fig.2. Methods to handle these singularities are well studied, for example, with the spline method [10] or the point method [11, 12]. In this work, we employ the point method.

2.2 Point method

The point method has been proposed by Schlessinger [11] and developed by Kamada *et al.* [12]. We evaluate the amplitudes $X_{i,j}(p_i, p_j, W)$ in Eq.(1) at $W = W' + i\epsilon_i$ with a real W' and a finite positive ϵ_i ($i = 1, 2, \dots$). With finite ϵ_i , the logarithmic singularities in $Z_{i,j}(p_i, p_j, W)$ become mild and numerical calculations can be performed safely. Then, we use the following continued fraction formula to extrapolate the amplitudes to the energy at $W = W' + i\epsilon$ with the infinitesimal positive ϵ :

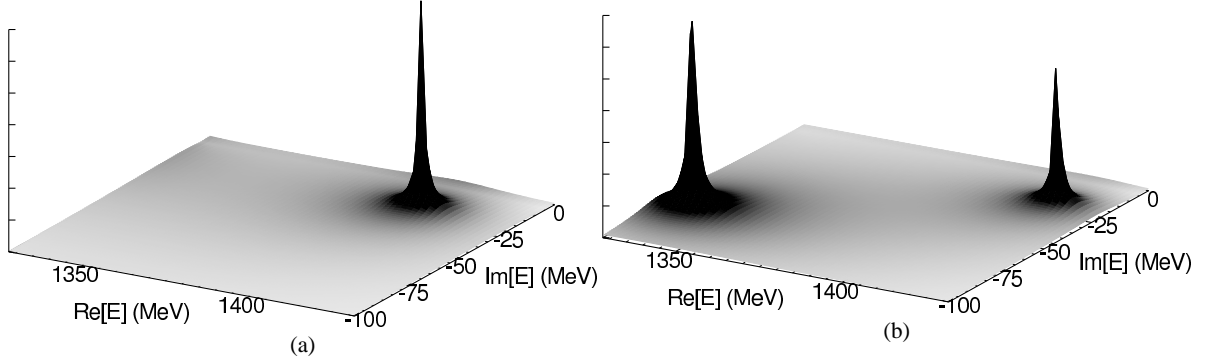
$$\begin{aligned} X(W' + i\epsilon) &= \frac{X(W' + i\epsilon_1)}{1 + \frac{a_1(\epsilon - \epsilon_1)}{1 + \dots}} \\ &= \frac{X(W' + i\epsilon_1)}{1 +} \frac{a_1(\epsilon - \epsilon_1)}{1 +} \frac{a_2(\epsilon - \epsilon_2)}{1 +} \dots, \end{aligned} \quad (4)$$

with

$$a_l = \frac{1}{\epsilon_l - \epsilon_{l+1}} \left(1 + \frac{a_{l-1}(\epsilon_{l+1} - \epsilon_{l-1})}{1 +} \frac{a_{l-2}(\epsilon_{l+1} - \epsilon_{l-1})}{1 +} \dots + \frac{a_1(\epsilon_{l+1} - \epsilon_1)}{1 - [X(W' + i\epsilon_1)/X(W' + i\epsilon_{l+1})]} \right). \quad (5)$$

Table 1 Cutoff parameters of $\bar{K}N - \pi\Sigma$ interaction.

	$\Lambda_{\bar{K}N}^{I=0}(\text{MeV})$	$\Lambda_{\pi\Sigma}^{I=0}(\text{MeV})$	$\Lambda_{\bar{K}N}^{I=1}(\text{MeV})$	$\Lambda_{\pi\Sigma}^{I=1}(\text{MeV})$
E-indep.	1000	700	920	960
E-dep.	1000	700	725	725

**Fig. 3** The $S = -1$, $J^\pi = 1/2^-$ $\bar{K}N$ s-wave amplitude on complex energy plane in (a) the E-indep. model and (b) the E-dep. model.

3 Model of Two-body Interaction

We employ the two models for the meson-baryon interaction of the $\bar{K}NN - \pi\Sigma N$ system. One is the model with the energy independent (E-indep.) separable potentials employed in [4]

$$V_{\alpha\beta}(q', q) = -\lambda_{\alpha\beta} \frac{1}{32\pi^2 F_\pi^2} \frac{m_\alpha + m_\beta}{\sqrt{m_\alpha m_\beta}} \left(\frac{\Lambda_\alpha^2}{q'^2 + \Lambda_\alpha^2} \right) \left(\frac{\Lambda_\beta^2}{q^2 + \Lambda_\beta^2} \right), \quad (6)$$

and another is the model with the energy dependent (E-dep.) potentials employed in [13]

$$V_{\alpha\beta}(q', q; E) = -\lambda_{\alpha\beta} \frac{1}{32\pi^2 F_\pi^2} \frac{2E - M_\alpha - M_\beta}{\sqrt{m_\alpha m_\beta}} \left(\frac{\Lambda_\alpha^2}{q'^2 + \Lambda_\alpha^2} \right) \left(\frac{\Lambda_\beta^2}{q^2 + \Lambda_\beta^2} \right). \quad (7)$$

Here, α and β specify the meson-baryon channels, m_α (M_α) is the meson (baryon) mass of the channel α ; q and q' are the relative momenta of the channels α and β in the center of mass system, respectively; F_π is the pion decay constant; E is the total scattering energy of the meson-baryon system, which is determined by $W - p^2/2\eta$ with η being the reduced mass between spectator particle and meson-baryon pair in the three-body system; $\lambda_{\alpha\beta}$ is determined by the flavor SU(3) structure of the Weinberg-Tomozawa term, assuming the different off-shell behavior with non-relativistic kinematics.¹ Also, we have introduced the cutoff parameter Λ_α . These parameters are determined by fitting the $\bar{K}N$ cross sections (The resulting values of the parameters are listed in Table 1). Here we take “non-relativistic kinematics” in this report.

We find that the above two models have a quite different analytic structure of the $\bar{K}N$ s-wave amplitude in the complex energy plane below the $\bar{K}N$ and above the $\pi\Sigma$ threshold energies: the E-indep. model has a pole corresponding to $\Lambda(1405)$ in the $\bar{K}N$ physical and $\pi\Sigma$ unphysical sheet (Fig. 3(a)), while the E-dep. model has two poles in the same sheet (Fig. 3(b)). The analytic structure of the E-dep. model is similar to that of the chiral unitary model [14]. It will be then interesting to examine how this difference between the models of the two-body interaction emerges in the strange-dibaryon production reactions.

4 Results and Discussion

In this report, we presents the quasi two-body amplitudes by using the most important interactions. For three-body Z, we include \bar{K} -exchange mechanism but not π or baryon exchange mechanism. For two-body interac-

¹ In deriving the potentials from the Weinberg-Tomozawa term, we have also assumed $E_\alpha/M_\alpha \sim 1$ for baryons.

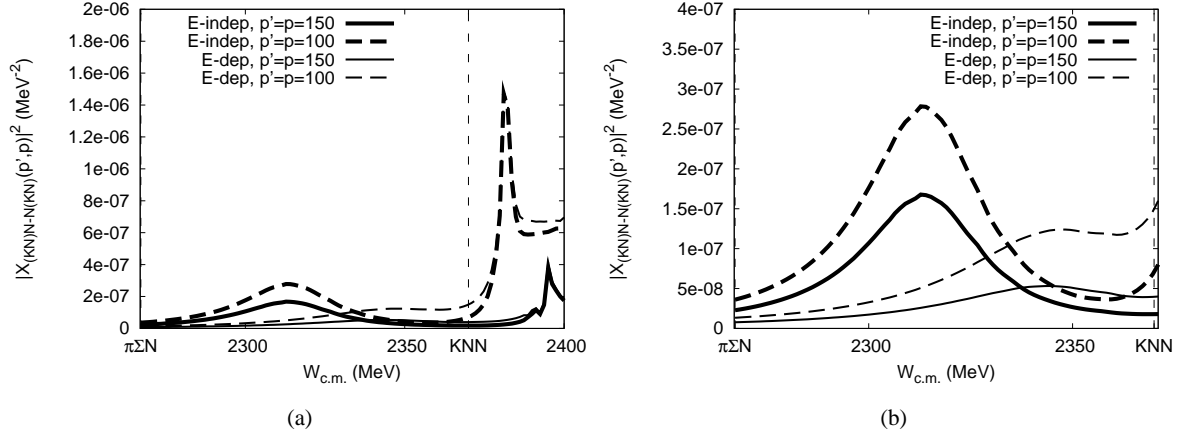


Fig. 4 Energy dependence of $|X_{(\bar{K}N)N-(\bar{K}N)N}(p, p_j)|^2$. Dashed lines represent the amplitude at 100 MeV, and solid lines represent the amplitude at 150 MeV. Thick lines are E-indep. model, and thin lines are E-dep. model. (b) is enlarged view of (a).

tion, we include $\bar{K}N - \pi\Sigma$ interaction. In Fig.4, we show $|X_{(\bar{K}N)N-(\bar{K}N)N}(p_i, p_j, W)|^2$ on the real energy axis for the E-indep. (thick curves) and E-dep. (thin curves). We observe a peak around $W \sim 2310$ MeV for the E-indep. model and a bump around $W \sim 2350$ MeV for the E-dep. model. These peak and bump appear near the calculated resonance energy of the strange-dibaryons ($W_R = 2329.5 - i23.3$ MeV for the E-indep. model and $W_R = 2352.0 - i22.5$ MeV for the E-dep. model). This result suggests that the signal of the strange-dibaryons can emerge as a clear peak or a bump of the cross sections, which can be calculated from the amplitude-square $|X|^2$ on the real energy axis. The peak structure is pronounced in the E-indep. model, while in the E-dep. model it is rather small and may not be possible to separate from the background contributions. This difference of the three-body amplitudes due to the model dependence of the two-body subsystem suggests that the strange-dibaryon production reactions could provide also the useful information on the $\bar{K}N - \pi\Sigma$ system.

In summary, by making use of the point method, we have calculated the quasi two-body amplitude $X_{i,j}(p_i, p_j, W)$ on the real energy axis. We then have found the bump structure in the amplitude in the energy region where the strange-dibaryons are expected to exist, implying that the signal of the strange-dibaryon resonances is possible to be observed in the physical cross sections. We have also shown that the strange-dibaryon production reactions could also be useful for judging existing dynamical models of $\bar{K}N - \pi\Sigma$ system with $\Lambda(1405)$. In the current work, however, we have not taken account of several reaction mechanisms such as π -exchanges. The further improvement of the current model and the calculation of the actual cross sections are under investigation.

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